Capacitance

A capacitor essentially consists of two conducting surfaces separated by a layer of an insulating medium called dielectric. Mathematically,
Capacitance $C = {}^{Q}/_{V} = {}^{\text{Charge}}/_{\text{Potential difference}}$ (Farad).

Parallel-plate capacitor

A parallel-plate capacitor consisting of two plates M and N each of area Am^2 separated by a thickness d meters of a medium of relative permittivity ε_r. If a charge of +Q coulomb is given to plate M, then flux passing through the medium is $\psi = Q$ coulomb.
Flux density in the medium $D = {}^{\psi}/_{A} = {}^{Q}/_{A}$
And electric intensity of the medium $E = {}^{V}/_{d}$
Hence, flux density is related as, $D = \varepsilon E$
So, ${}^{Q}/_{A} = \varepsilon {}^{V}/_{d}$
Or, ${}^{Q}/_{V} = \varepsilon {}^{A}/_{d}$
Or, $C = {}^{\varepsilon A}/_{d}$

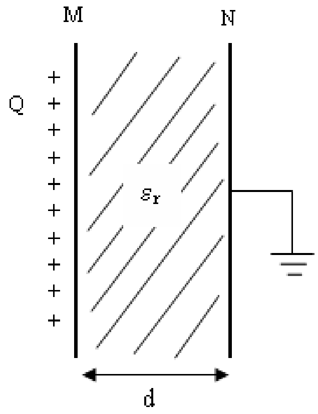


Figure: Capacitor

Capacitors do not have a stable "resistance" as conductors do. However, there is a definite mathematical relationship between voltage and current for a capacitor, as follows:

"Ohm's Law" for a capacitor

$$i = C\frac{dv}{dt}$$

Where,

i = Instantaneous current through the capacitor
C = Capacitance in Farads
$\frac{dv}{dt}$ = Instantaneous rate of voltage change (volts per second)

Current-voltage relationship in a capacitor

The charge on capacitor is given by the expression, $Q = CV$
Differentiating this relation we get,

$$\frac{d}{dt}(Q) = \frac{d}{dt}(CV)$$

$$or, \quad i = C\frac{dV}{dt}$$

Again,

$$dV = \frac{i}{C}dt$$

$$or, \quad \int dV = \int \frac{i}{C}dt$$

$$or, \quad V = \frac{1}{C}\int_{o}^{t} idt$$

Capacitors in series and parallel

$1/C_{total} = 1/C_1 + 1/C_2 + 1/C_3 + \ldots + 1/C_N$

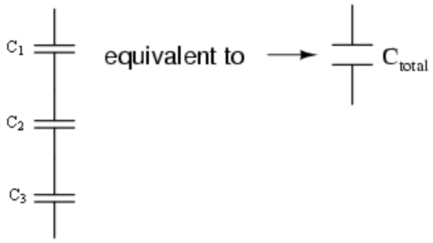


Figure: Capacitors in series

$C_{total} = C_1 + C_2 + C_3 + \ldots + C_N$

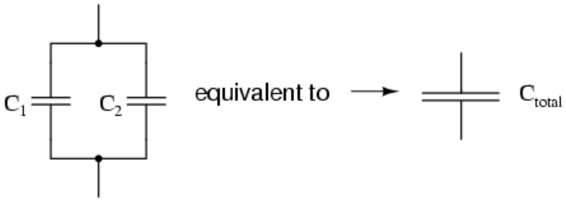
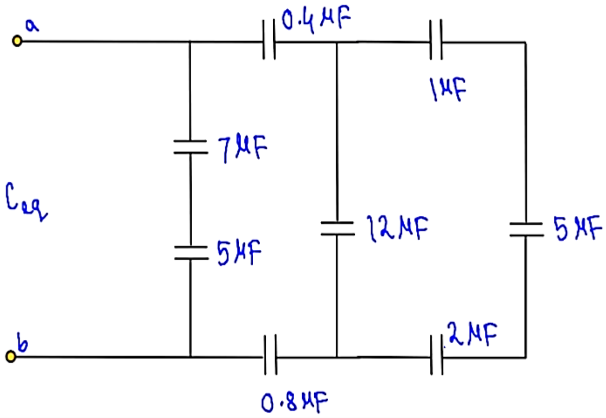


Figure: Capacitors in parallel

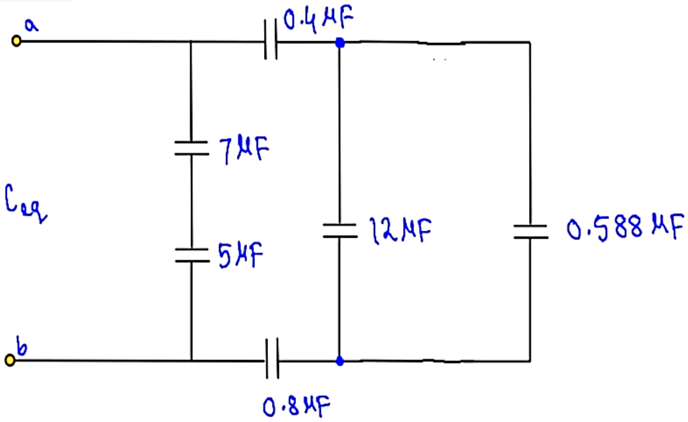
Example: Determine the equivalent capacitance for the circuit shown below.

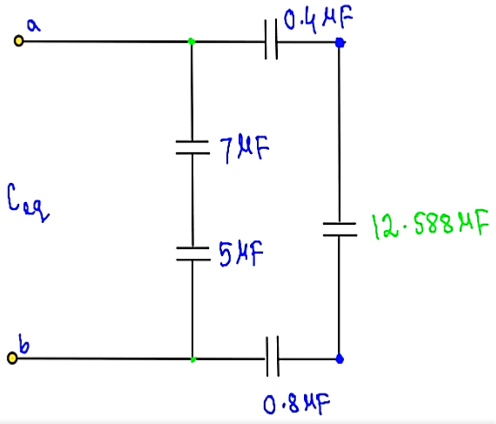
Solution:



$$\frac{1}{C_{eq_1}} = \frac{1}{1\mu F} + \frac{1}{5\mu F} + \frac{1}{2\mu F}$$

$$\Rightarrow C_{eq_1} = 0.588\mu F$$

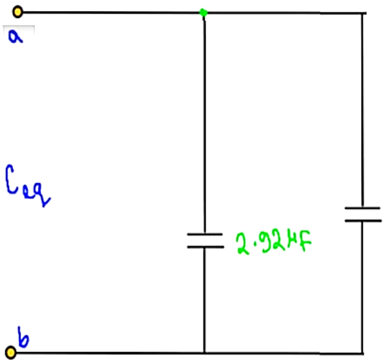




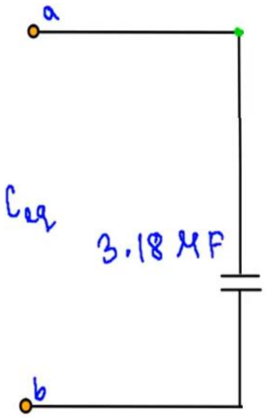
$$\frac{1}{C_{eq_3}} = \frac{1}{0.4\mu F} + \frac{1}{12.58\mu} + \frac{1}{0.8\mu F}$$

$$\Rightarrow C_{eq_3} = 0.261\ \mu F$$

$$C_{eq_4} = \frac{7\mu F \times 5\mu F}{7\mu F + 5\mu F} = 2.92\ \mu F$$

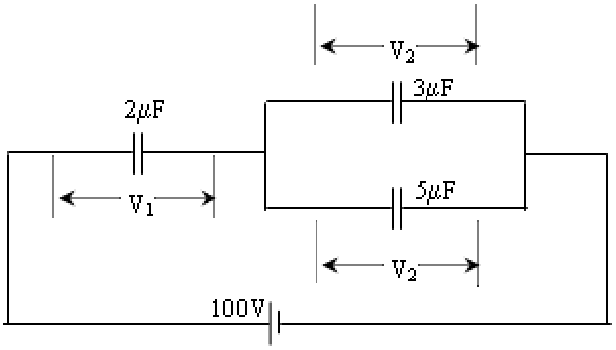


$$C_{eq} = 2.92\ \mu F + 0.261\ \mu F$$
$$= 3.18\ \mu F$$



Example:

Find the charges on capacitors of the fig. given below & the potential difference across them.



Solution:

The equivalent capacitance $C_{eq} = C_1 \mid\mid (C_2 + C_2) = 2 \mid\mid (3 + 5) = 2 \mid\mid 8 = 1.6\ \mu F$
Now,
$Q = CV$
Or, $Q = Q_1 = 1.6\ \mu F \times 100\ V = 160\ \mu Coul$
So we get, $V_1 = {}^{Q1}/_{C1} = {}^{160}/_{2} = 80\ V$
$\therefore\ V_2 = 100 - 80 = 20\ V$
$Q_2 = C_2V_2 = 3 \times 20 = 60\ \mu Coul$

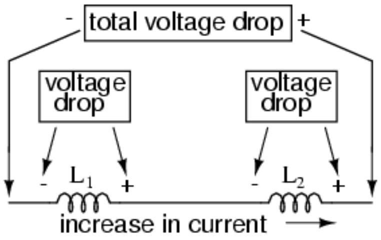
$Q_3 = C_3V_2 = 5 \times 20 = 100\ \mu Coul$

Inductance

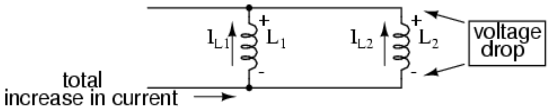
Inductance is a circuit property that is due entirely to the magnetic field created by current in a circuit. The effect that inductance has on circuit operation is to oppose any change in current. A circuit element built to possess inductance is called an inductor. In its simplest form an inductor is simply a coil of wire. Ideally, inductors have only inductance. However, since they are made of wire, practical inductors also have some resistance. The unit of inductance is the Henry.

Inductors in series and parallel

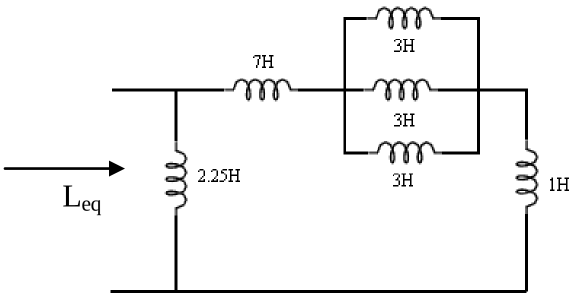
$L_{total} = L_1 + L_2 + L_3 + \ldots + L_N$



${}^1/L_{total} = {}^1/L_1 + {}^1/L_2 + {}^1/L_3 + \ldots + {}^1/L_N$



Example: Determine the equivalent inductance for the circuit shown below.



Solution:

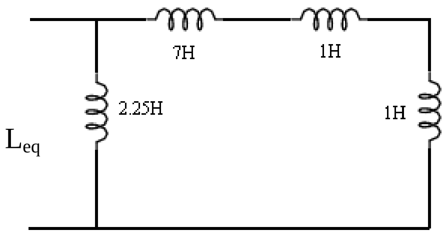


Figure: (a)

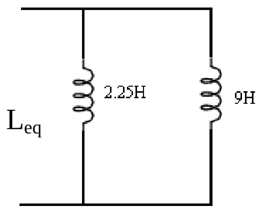


Figure: (b)

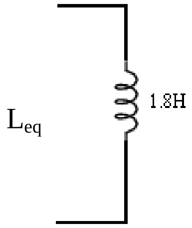


Figure: (c)